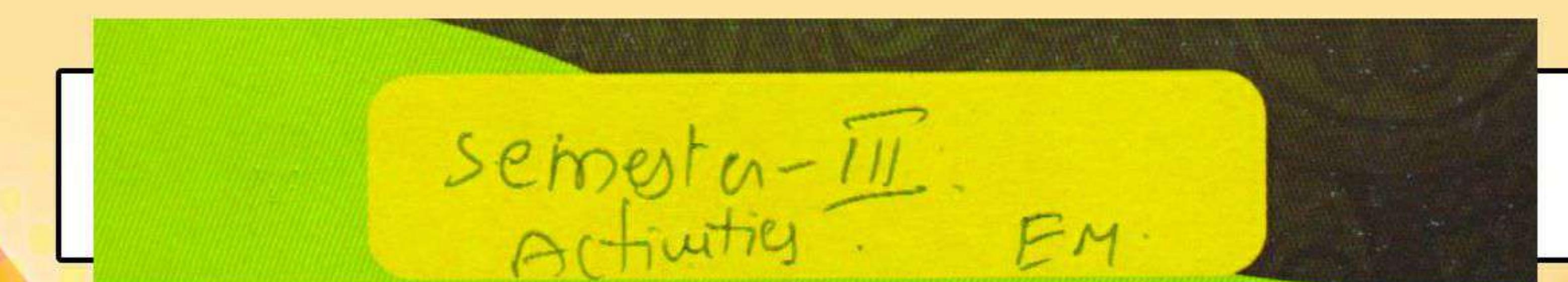




# MOTHER COLLEGE OF EDUCATION (B.Ed & D.Ed)

(Approved by NCTE & Affiliated to SCERT, ANU & Govt. of A.P.)

DARSI - 523 247, Prakasam District, A.P.



Name .....
Subject : PEDAGOGY OF MATHEMATICS
Roll No. ....
Register No. ....

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**20 - 20**

7) proportion — Three red sweets and two blue. Ask what proportion of the sweets is blue. How many sweets will say  $\frac{2}{3}$  rather than  $\frac{2}{5}$ ? Why? Because there are comparing blue to red, not blue to all the sweets. Always stress that proportion is "part to whole".

8) perimeter and area confuse many students! —

→ A common mistake, when measuring the perimeter of rectangle, is to count the squares surrounding the shape. In the same way as counting those circles for area. Now you can see why some would give the perimeter of a two by three rectangles as 14 units rather than 10.

9) misreadings scales - still identified as a weakness! —

In key stage test papers. The most common misunderstanding is that any interval on a scale must correspond to one unit. Subsequent handling of different scales divided up in to twos, fives, tens, tenths, etc. will help to banish the idea.

A definition of conceptual understanding on right on the confusion about conceptual understanding and the pressing problem of students misunderstandings. I think a slightly more definition of conceptual understanding is wanted. I prefer to define it thus we conceptual understanding in mathematics means that students understand which ideas are key and that the

## -:- ACTIVITY-:- 1

Create different activities to realize concept attainment by children in any unit from mathematics :—

### CONCEPTUAL UNDERSTANDING IN MATHEMATICS :-

The Common Core Standards in mathematics stress the importance of conceptual understanding as a key component of mathematical expertise. I find in my experience, many math teachers do not understand conceptual understanding for two main reasons that if students know all the definitions and rules, then they possess such understanding.

Students understand connection between counting and addition and subtraction. They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on. These properties help solve addition and subtraction problems with ease. By comparing a variety of solution strategies children build their understanding of the relationship between addition and subtraction.

Note that I highlighted understandings requires focused inferential work. Being helped to generalize to genuine understanding.

Knowledge of procedures is no guarantee of conceptual understanding. For example, many children can execute a procedure to divide fractions without understanding why the procedure works. Most observers agree that knowledge of procedures, and concept is desirable.

another common conceptual problem understanding that an equal sign (=) refers to equality, that is mathematical equivalence by some estimates as few as 25 percent of American 6th graders have a deep understanding of this concept. Students often think it signifies "put the answer here".

1) A number with three digits is always bigger than one with two. Some children will swear blind that  $3.24$  is bigger than  $4.6$  because it's got more digits. Why? Because for the first few years of learning, they only come across whole numbers where the digits rule does work.

2) When you multiply two numbers together, the answer is always bigger than both the original numbers. Another seductive 'rule' that works for whole numbers but falls to pieces when one or both of the numbers is less than one. Remember that. Instead of the word 'times' we can always substitute the word of. So  $\frac{1}{2}$  times  $\frac{1}{4}$  is the same as a half of a quarter. That immediately demolishes the expectation that the product is going to be bigger than both original numbers.

3) Which fraction is bigger  $\frac{1}{3}$  or  $\frac{1}{6}$ ?

How many pupils will say  $\frac{1}{6}$  because they know that 6 is bigger than 3? This reveals a gap in knowledge about what the bottom numbers, the denominator, of a fraction does, if divides the top numbers. The oxymoron of course practical wrong such as cutting pre-divided cheese onto thirds and concept is desirable.

Another common conceptual problem is understanding that an equal sign means equals and comparing the shapes helps the cement understanding of fractions.

4) Common regular shapes aren't recognised for what they are unless they are upright. Teachers can inadvertently feed this misconception if they always draw a square upright and they are 'isosceles triangle in the 'usual' position. Why not draw them occasionally upside down, facing a different direction, or just tilted over, to force pupils to look at the essential properties and by the way, in maths, there is no such thing as a diamond. It is either a square or a rhombus.

5) The diagonal of a square is the same length as the side -

Not true, but tempting for many young minds, so, how about challenging the class to investigate this by drawing and measuring. Once the top table have mastered this, why not ask them to estimate the dimensions of a square whose diagonal is exactly 5cm. Then draw it and see how close their guess was.

6) To multiply by 10, just add a zero. Not always what about  $23.7 \times 10$ ,  $0.35 \times 10$  or  $\frac{2}{3} \times 10$ ? Try to spot and unpack the just and zero rule whenever it rears its head.

grasp the heuristic value of those idea. They are thus better able to use them strategically to solve problems especially non-routine problems and avoid common misunderstandings as well as inflexible knowledge and skill.

In other words, students demonstrate understandings of -

- 1) which mathematical ideas are key, and why they are important
- 2) which ideas are helpful in a particular context for problem solving.
- 3) why and how key ideas aid in problems solving by reminding us of the systematic nature of mathematics
- 4) How an idea or procedure is mathematically defensible why we and they are justified in using it.
- 5) how to flexibly adapt previous experience to new transfer problems.

### A test for Conceptual understanding

Rather than explain my definition further here, I will operationalize it in a little test of 13 questions to be given to 10<sup>th</sup>, 11<sup>th</sup>, and 12<sup>th</sup> grader who have passed all traditional maths courses through algebra and geometry (middle school students can be given the first questions. Math teacher, give it to your students tell us the results.

I will make a friendly wager, I predict that no student will get all the questions correct, prove me wrong and I shall give the teacher and students a big shout-out.

- 1) "you can't divide by zero" explain why not (even though of course you can multiply by zero)
- 2) Solving problems typically requires finding equivalent statements that simplify the problem "explain" and in so doing refine the meaning of the = sign.
- 3) you are told to "invert and multiply" to solve division problems with fractions. But why does it work? prove it.
- 4) place these numbers in order of largest to smallest.  
 $0.0152, \frac{1}{66}, 0.015, .001, .002$ .
- 5) multiplication is just repeated addition "explain" why this statement is false, giving examples.
- 6) A Catering Company rents out tables for big parties & people can sit around a table. A School is giving a party for parents, sibling, students and teacher. The guest list total 243. How many tables should the school rent.
- 7) most teachers assign final grades by using the mathematical mean, to determine them. Give at least 3

## S-YTIVITJA

Reasons why the mean may not be the best measure of achievement by explaining what the mean hides.

- 8) Construct a mathematical equation that describes the mathematical relationships between feed and yards. Hint all you need as parts of the equation are  $x$ ,  $y$  and 3
- 9) As you know, PEMDAS is shorthand for the order of operation for evaluating complex expressions. The order of operations is a convention  $x(A+B) = xA + xB$  is the distributive property. It is a law. What is the difference between a convention and a law, then? give another example of each.
- 10) Why were imaginary numbers invented?
- 11) What is the difference between an "accurate" answer and an "appropriately precise" answer?

## ACTIVITY-2

### 2. INDIAN MATHEMATICIANS AND THEIR CONTRIBUTIONS:-

#### RAMANUJAN :-

- He was born on 22 of December 1887 in a small village Nantore district madras. He failed in English intermediate, so his formal studies were stopped but his study of mathematics continued.
- He sent a set of 120 theorems to professor Hardy of Cambridge. As a result he invited Ramanujan to England.
- Ramanujan showed that any big number can be written as a sum of not more than four prime numbers.
- He showed that how to divide the number into two or more squares or cubes.
- When Mr. Littlewood come to see Ramanujan on taxi number 1729. Ramanujan said that 1729 is the smallest number which can be written on the form of sum of cubes of two numbers in two ways i.e.  
$$1729 = 9^3 + 10^3 = 1^3 + 12^3$$
 Since then the number 1729 is called Ramanujan's number.
- On the third century B.C. Archimedes noted that the ratio of circumference of a circle to its diameter is constant. The ratio is now called

'pi' ( $\pi$ ) (The 16th letter in the greek alphabet series)

→ The largest numbers the Greeks and the Romans used were  $10^6$  whereas Hindus used numbers as big as  $10^{53}$  with specific names as early as 500BC during the Vedic period.

### ARYABHATA :-

→ Aryabhata was born in 476 A.D. in Kusumapura, India.

→ He was the first person to say that earth is spherical and it revolves around the sun.

→ He gave the formula  $(a+b)^2 = a^2 + b^2 + 2ab$

→ He taught the method of solving the following problems.

$$1+2+3+4+\dots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+4^2+5^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+4^3+5^3+\dots+n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

$$1^4+2^4+3^4+4^4+5^4+\dots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n+1)}{30}$$

### BRAHMAGUPTA :-

→ Brahmagupta was born in 598 A.D. in Panjab, India.

→ He gave fair methods of multiplication.

→ He gave the following formulae used in GP Series

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = (ar^{n-1}) \frac{(r^n - 1)}{r - 1}$$

→ He gave the following formula - Area

Area of cyclic quadrilateral with side  $a, b, c, d$

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}, \text{ where } s = \frac{a+b+c+d}{2}$$

$$\text{Length of its diagonal} = \sqrt{\frac{bc+ad}{ab+cd}} (ac+bd) \times \sqrt{\frac{ab+cd}{bc+ad}} \times \frac{ab+cd}{bc+ad} (ac+bd)$$

### SHAKUNTALA DEVI:

- She was born in 1939
- In 1980 she gave the product of two, thirteen digit numbers with in 28 seconds, many countries have invited her to demonstrate her extra ordinary talent
- In Dallas she competed with a computer to see who gave the cube root of 188138517 faster. She was at University of USA. She was asked to give the 23rd root of 916748676920039158098660927585380148310662014430862240712651642793365704086749659.
- She answered in 50 seconds. The answer is 54637891. It took a UNIVAC 108 Computer, full one minute (10 seconds more) to confirm that she was right after it was fed with 13000
- Now she is known to be human Computer.

## BHASKARACHARYA :-

- He was born in a village of Mysore district
- He was the first to give that any number divided by '0' gives infinity (so)
- He written a lot about zero, seeds, permutation and combination.
- He wrote "The hundredth part of the circumference of a circle seems to be straight. our Earth is a big sphere and that's why it appears to be flat."
- He gave the formulae large  $\sin(A+B) = \sin A \cos B + \cos A \sin B$ .

## ACTIVITY-3

preparation of T.L.M for any one topic from class 6-10 mathematics

Mathematics Teaching learning material (T.L.M) for primary and middle school classes

In primary and middle school classes there are many concepts in mathematics related to our environment. It is very nice to arrange the teaching learning materials for those classes. Using teaching learning material is very useful to make teachers presentation easy effective and attractive. Teacher learning material makes the mathematical concept easy and interesting. Let's talk about some geometrical terms and related T.L.M that can be used and prepared easily in any school. There may be different innovative ideas smart math learning. How to make math learning smart and interesting?

On first article there are my own idea. What are your experience? Please share with first blog. If you want to get my books, please like my google plus page

"innovative education" to know more please visit + page "about first Blog".

useful teaching learning materials (T.L.M) for mathematics teaching in middle school Classes

Teaching learning material is the great tool to teach the students. It simplifies the subjects and makes teachers presentation interesting and attractive. Students learn better by examples and T.L.M. In an innovative classroom just like smart class there are many opportunities for a teacher to arrange many teaching learning materials online. To know more about some useful and attractive teaching learning techniques visit other posts on this blog "Innovative Education".

### Ready Made - Teaching Learning Materials :-

In many schools there are many ready made T.L.M.s are available for teachers. Many schools have science and mathematics kits. They can purchase several ready made teaching learning materials from market. This ready made T.L.M. is very useful. Teachers can purchase many mathematical instruments for classroom presentation. These instruments and equipment are very helpful for teacher and student.

### Mathematics Kit for Class :-

It is good idea to prepare "mathematics kit" in school. A mathematics kit should contain essential equipment used in classroom teaching. It may contain various models of maths instruments and equipment's like scale, ruler, compass, geometry box, number lines model and

such equipment. It should be easily available for students and teachers should encourage the students to use this kit. In a primary school teacher's have many ideas for teaching learning materials like ball, pencils, seeds etc. These are very cheaper and easily available. There should be a separate box to put those learning materials.

### on line learning Resources for Mathematics

There are several online tools for teaching mathematics.

Teachers can find many awe some websites useful in their classroom. Even they can find many sites teach how to teach numbers, How now students can learn by social networking websites. Many social and educational websites are providing a facility to share educational content and ideas world wide. The use of such websites is very helpful for teachers and students.

Different learning software helps the teachers to make mathematics learning easy and funny. Teacher can

collaborative learning techniques within the class.

It is very interesting to introduced project based learning. It is very helpful to use audio visual just like learning videos, to explain the different hard topics in an easy way. Teacher can use different types of mathematics online learning games as powerful teaching material.

( Teaching Learning

The Geometrical concepts in primary and middle school. :-

In primary and middle school syllabus in madhya pradesh (andhra) there are some basic concept in mathematics like line rectangle, square, triangle, circle, field, cone, perimeter and such type basic concepts. The textbooks are designed in such type to correlate these concepts with the environment. use of sketch pens and colour pencils are very useful to create the interest and attraction among the students in mathematics learning.

### Easy T.L.M. That may be used by teacher:

It is very easy to use teaching learning materials in these classes. Teacher can use different type easily available teaching learning materials as match box for rectangle, scale for line. There are many teaching learning materials can be prepared by teachers using easily available materials. The can make different learning materials by card-sheets, wood, wire bangles, match boxes and so many materials easily available in environment. It is a wise idea to prepare a cloth sheet for numbers 1, 2, 3 when teacher uses collaborative techniques to make the teaching learning materials in classroom, he can achieve many goals. An innovative teacher has a lot of innovative ideas for T.L.M. Just try to use T.L.M. in your classroom and bring the new experiences of mathematics learning with joy.

How to prepare mathematical teaching learning materials in class?

It is an easy and interesting activity to prepare mathematical T.L.M and models in classroom. Teacher can provide it as a project for the groups of students. Teacher should facilitate the students to prepare the projects. Teacher can arrange the essential materials for students when students learn how to prepare the T.L.M they inspired to do such activities again and again.

White board as powerful Teaching learning Materials

Teachers use white board in teaching the classroom. What are the smart and innovative ways to use white board as effective teaching learning materials in mathematics teaching? To smart and active teaching learning methodologies inspire the teachers to use more and more interactive teaching style. These teaching methodologies ensure the students' participation in learning. Teacher can facilitate the students to use white board to explained their problems, ideas, and practices. When students use white board they can learn better and they will be able to remember the concepts for long time. Teacher can plan how to use the white board for students. He can use white board as interactive teaching learning material. Many educational websites are providing learning games and multimedia.

on mathematics. Teachers can display these multimedia lessons on white boards by the computers and projectors.

## Smart and innovative Teaching Learning materials for smart classes ! —

In modern class room where we are planning to use internet and multimedia devices to make classroom learning easy, interactive and interesting. It is very nice to use internet based teaching learning materials to make mathematics learning Smart and innovative. What is the internet based learning materials? Internet provides several opportunities to make teaching interesting and interactive online learning games are the best example for this. There are many learning games for students to make mathematics learning easy and interesting. Smart school class is very suitable for using these learning games. Better students learn better by games and interactive teaching techniques.

### ACTIVITIES - 4

- Preparation of power point presentation (PPP) for any one topic from a different branch of mathematics.

## Some applications on Trigonometry :-

What is trigonometry ?

Trigonometry is a branch of mathematics that studies triangles and the relationships between their sides and angles between these sides.

### LINE OF SIGHT

We observe generally that children usually look up to see an aeroplane when it passes overhead thus line joining their eyes to the plane, while looking up is called line of sight.

### Angle of Elevation :-

The angle which the line of sight makes with a horizontal line drawn away from their eyes is called the angle of elevation of aeroplane from them. In this topic we shall make use of trigonometric ratios to find the height of tree, a tower, a water tank, width of a river, distance of ship from light house etc.

### Angle of Depression :-

If the pilot of aeroplane looks downwards at any object on the ground then the angle between his line of sight and horizontal line drawn away from his eye is called angle of depression.

Now let us solve some problem related to height and distance

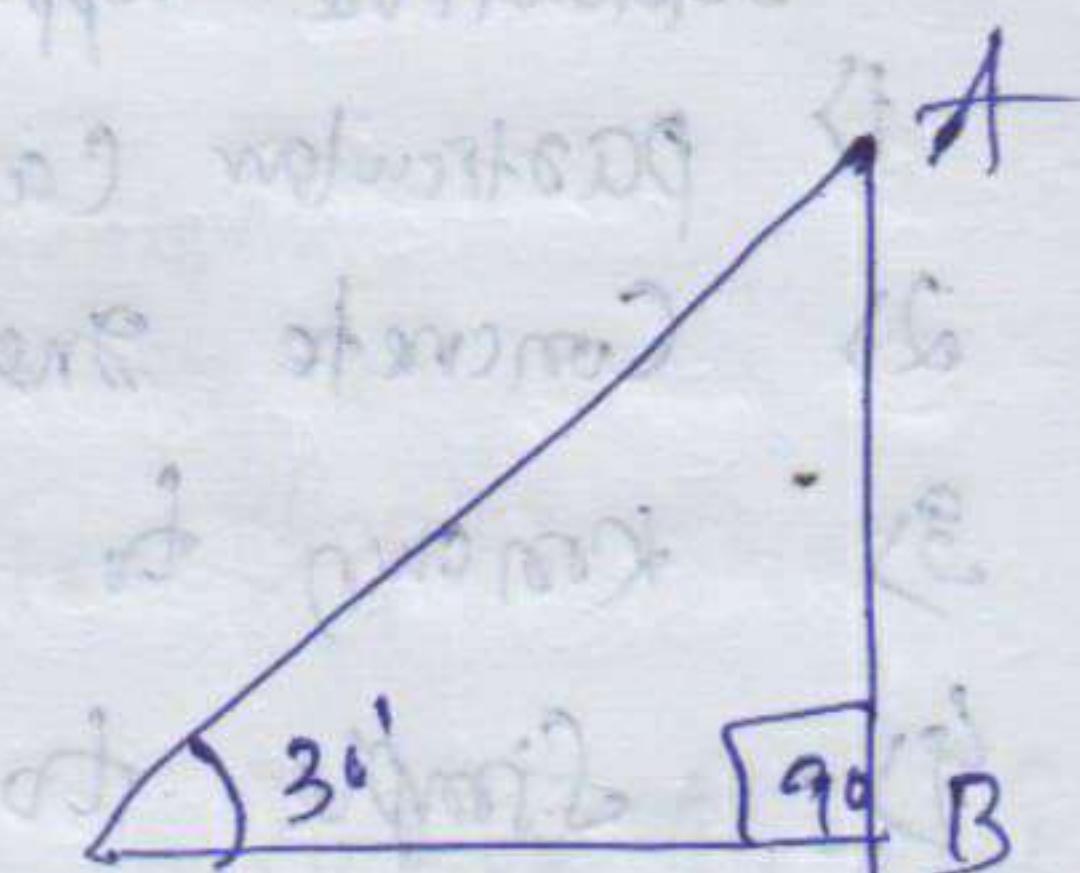
The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower is  $30^\circ$ . Find the height of the tower.

Ans Let AB be the tower and angle of elevation from point 'C' (on ground) is  $30^\circ$ .

$$\text{In } \triangle ABC, \frac{AB}{BC} = \tan 30^\circ$$

$$\Rightarrow \frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$



$\therefore$  the height of the tower is  $10\sqrt{3} \text{ m}$

## ACTIVITIES - 5

### METHODS OF TEACHING MATHEMATICS :-

#### INDUCTIVE METHODS

- Inductive approach is advocated by Pestalozzi and Francis Bacon
- Inductive approach is based on the process of induction.
- Thus we first take a few examples and then generalize
- It is method of constructing a formula with the help of a sufficient number of concrete examples. Induction means to provide a universal truth by showing.

that if it is true for a particular case, it is true for all such cases. Inductive approach is psychological in nature.  
→ The children follow the subject matter with great interest and understanding. This method is more useful in arithmetic teaching and learning.

Inductive approach proceeds from:

- 1) particular cases to general rules or formulae
- 2) Concrete instance to abstract rules
- 3) Known to unknown
- 4) Simple to Complex

following steps are used while teaching by this method:

#### (a) PRESENTATION OF EXAMPLES:

→ In two step teacher presents many examples of same types and solutions of those specific examples given obtained with the help of the student.

#### (b) Observation:

After getting the solution the students observe and try to reach to some conclusion.

#### (c) Generalization:

After observation the examples presented, the teacher and children decide some common formulae, principles or laws by logical mutual discussion.

#### (d) Testing and verification:

After deciding some common formula, principle or law, children test and verify the law with the help of other examples.

### Demerits :-

- certain complex and complicated formula can't be generated so this method is limited in range and not suitable for all topics.
- it is time consuming and labours method.
- it is lengthy.
- its application is limited to very few topics.
- it is not suitable for higher classes.
- inductive reasoning is not absolutely conclusive because the generalization made with the help of a few specific examples may not hold good in all cases.

### Applicability of Inductive method

Inductive approach is most suitable where

- Rules are to be formulated
- Definitions are to be formulated
- formulate are to be derived
- Generalizations or laws are to be arrived at.

Example law of indices  $a^m \times a^n = a^{m+n}$

soln - we have starts with  $a^2 \times a^3 = (axa) \times (axaxa)$

$$\therefore a^5 = a^2 + 3$$

$$a^3 \times a^4 = (axaxa) \times (axaxaxa) = a^7 = a^{3+4}$$

$$\therefore a^m \times a^n = (\text{axaxa} \dots m \text{ times}) \times (\text{axaxa} \dots n \text{ times})$$
$$\therefore a^m \times a^n = a^{m+n}.$$

## MERITS :-

- It enhances self confidence.
- It is a psychological method.
- It is a meaningful learning.
- It is a scientific method.
- It develops scientific attitude.
- It develops the habit of intelligent hard work.
- It helps in understanding because the student knows now a particular formulae has been framed.
- Since it is a logical method, so it suits teaching Mathematics.
- It is a natural method of making discoveries majority of discoveries have been made intuitively.